4.2 Low genus surfaces

In this section we present several examples of low genus surfaces of different symmetry: BFY, CPD, GP, DDU, GM. Very interesting are the BFY and CPD surface. They are of the same symmetry as the Schwarz P surface. The BFY surface can be generated from an arbitrary initial configuration if the length of the unit cell is set close to the equilibrium length, i.e. the length for the minimum of the functional (2.9). The pattern cut out on the face of the unit cell for BFY surface resembles "butterfly" wings. That is why we have given the symbol BFY to this surface as a abbreviation for "butterfly". It is remarkable that the volume fraction for BFY structure is 0.5.

The volume fraction for the CPD is not equal to 0.5, as it is in the case of I-WP and O,C-TO structures. CPD surface is very similar to the minimal surface discovered by Neovius [39, 32]. The difference is such that CPD surface has a hole inside a unit cell, therefore its genus is bigger. The histograms of the mean curvature for BFY and CPD surfaces suggest that they can be minimal, Figs 4.26, 4.30.

Very interesting histogram has the GP surface, Fig. 4.35. This is the structure of gyroid phase symmetry. The histogram of the mean curvature for GP surface is peaked at $H = 0$ as it is for minimal surfaces, but it has also two symmetric peaks at $H \neq 0$. It rather cannot be attributed to the numerical errors, because this surface has low genus ($g = 21$) and thus should be calculated with high accuracy.

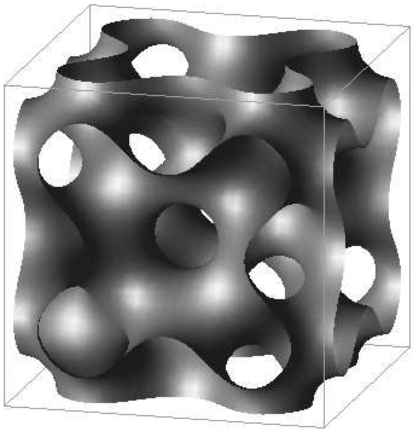

Figure 4.24: BFY surface generated from the functional (2.9). The unit cell. Off front view.

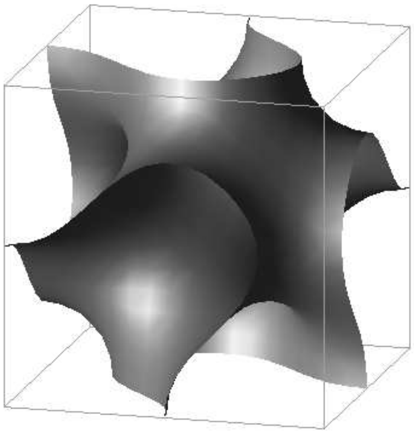

Figure 4.25: BFY surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

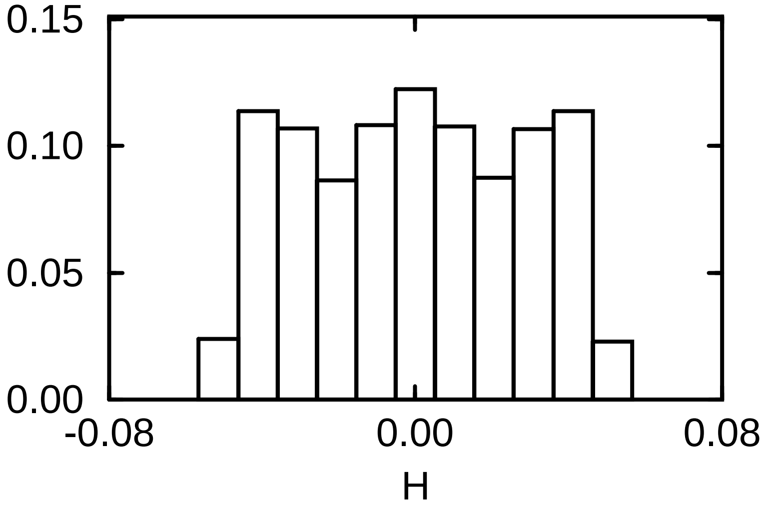

Figure 4.26: The histogram of the mean curvature for the surface BFY

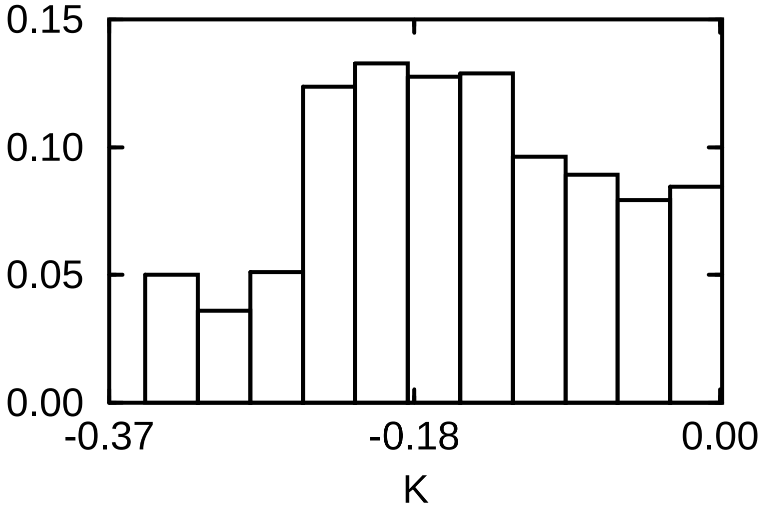

Figure 4.27: The histogram of the Gaussian curvature for the surface BFY

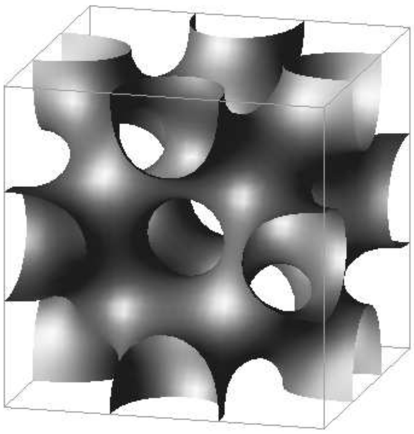

Figure 4.28: CPD surface generated from the functional (2.9). The unit cell. Off front view.

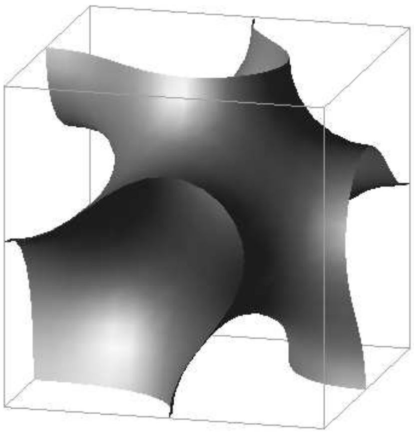

Figure 4.29: CPD surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

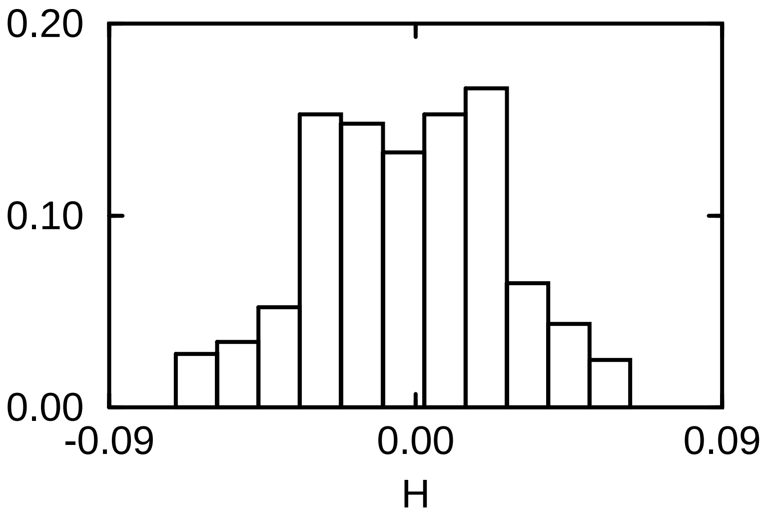

Figure 4.30: The histogram of the mean curvature for the surface CPD

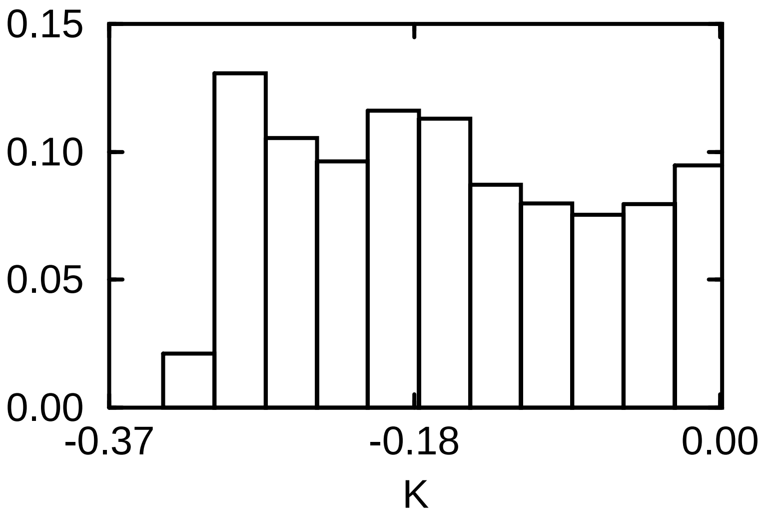

Figure 4.31: The histogram of the Gaussian curvature for the surface CPD

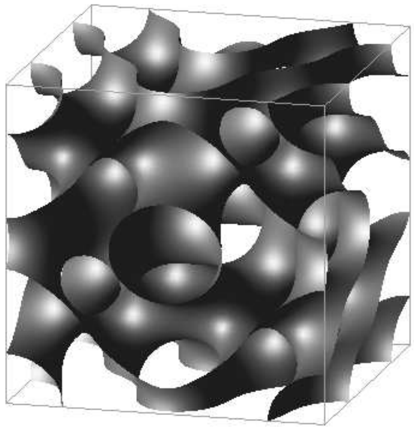

Figure 4.32: GP surface generated from the functional (2.9). The unit cell. Off front view.

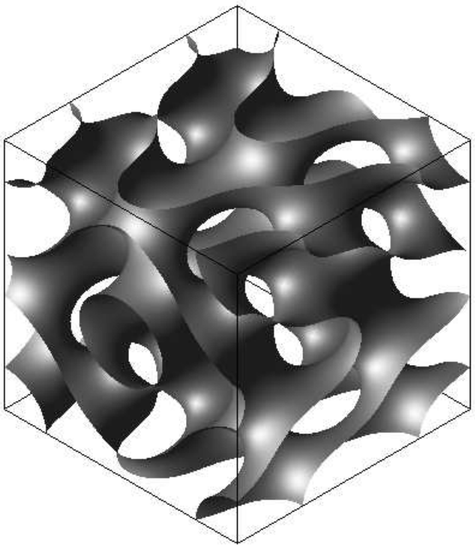

Figure 4.33: GP surface generated from the functional (2.9). The unit cell. Off diagonal view.

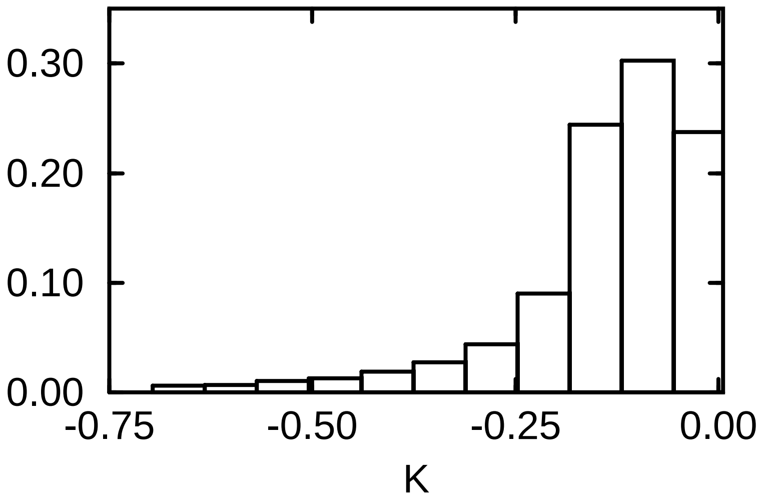

Figure 4.34: The histogram of the Gaussian curvature for the surface GP

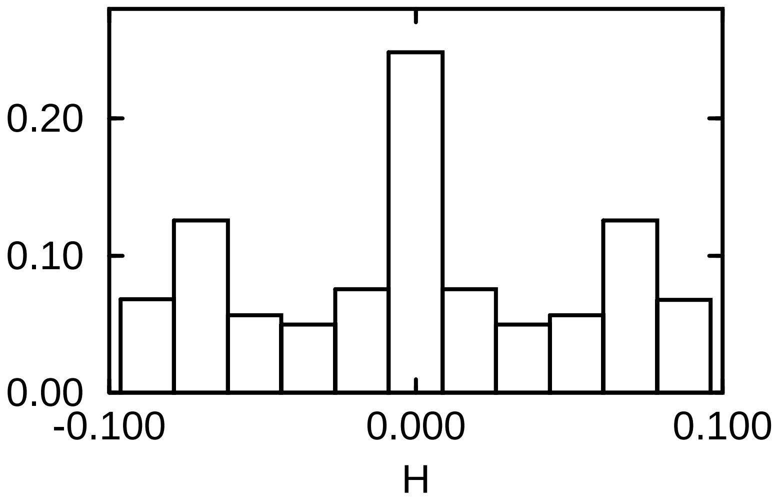

Figure 4.35: The histogram of the mean curvature for the surface GP

The structures GM and DDU are generated of trirectangular tetrahedron, see Fig. 3.13, i.e. in a similar way as the double diamond phase. The difference is such that 1/8 of the unit cell for the double diamond phase is a unit cell for the structures DDU and GM. The topology of the DDU structure is similar to the topology of the double diamond D structure. It can be seen by comparing the unit cell for DDU, Fig. 4.37, and the middle element of the double diamond structure D, Fig. 4.4. Thus it is rather unlikely that the DDU surface is minimal. This is also confirmed by the histogram of the mean curvature, Fig. 4.40. Although the DDU surface is not minimal it does not mean that the DDU structure cannot be found in real systems. In the scattering experiments one can investigate the symmetries of the structures, and on such basis one can guess the properties of the surfaces found in a given structure. The DDU structure was not known so far, and nobody considered its existence in real systems.

The GM structure is a combination of the Schwarz P surface, Fig. 4.8, and DDU surface. This surface as the DDU one also should not be minimal, see Fig. 4.46 The structures DDU and GM are not symmetric, i.e the volume fraction of oil and water is not equal to 0.5. The cubic cell for GM and DDU contains a few not connected pieces of the surface $\phi(\mathbf{r}) = 0$. However the whole surface is continuous. We have collected 8 cubic unit cells and shown in Figs. 4.38, 4.39, 4.44, 4.45 to illustrate how the infinite continuous triply periodic surface is built of the cubic unit cell.

Whether new surfaces we have generated are minimal or not is an interesting question. On the basis of numerical analysis we can speculate that

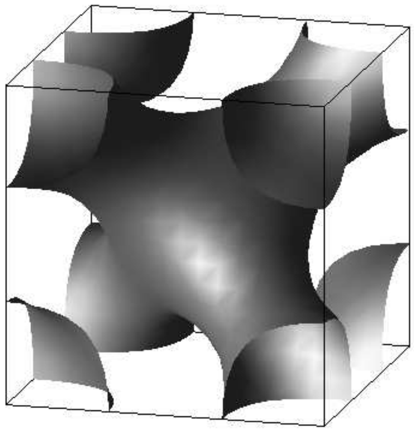

Figure 4.36: DDU surface generated from the functional (2.9). The unit cell. Off front view.

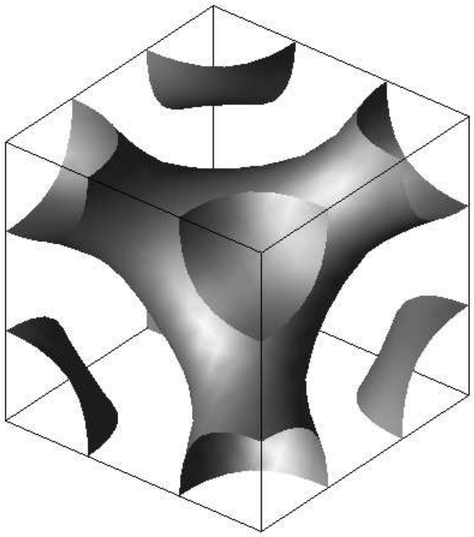

Figure 4.37: DDU surface generated from the functional (2.9). The unit cell. Off diagonal view.

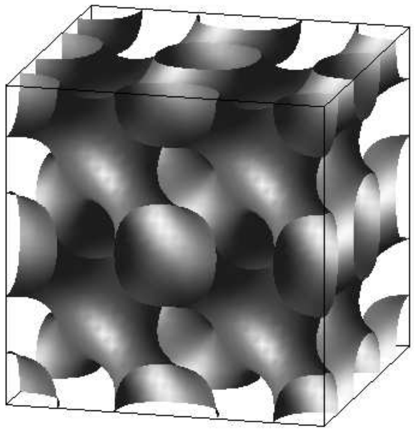

Figure 4.38: DDU surface generated from the functional (2.9). 8 unit cells. Off front view.

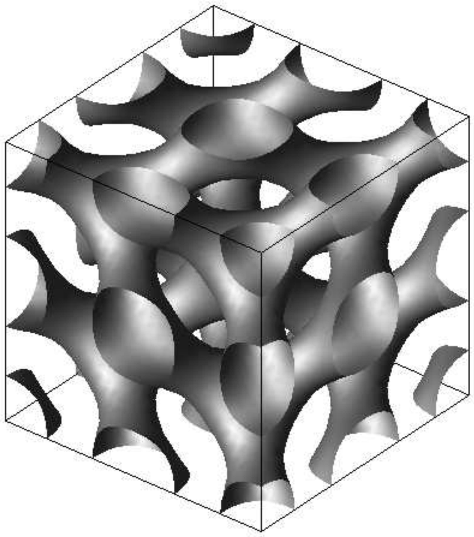

Figure 4.39: DDU surface generated from the functional (2.9). 8 unit cell. Off diagonal view.

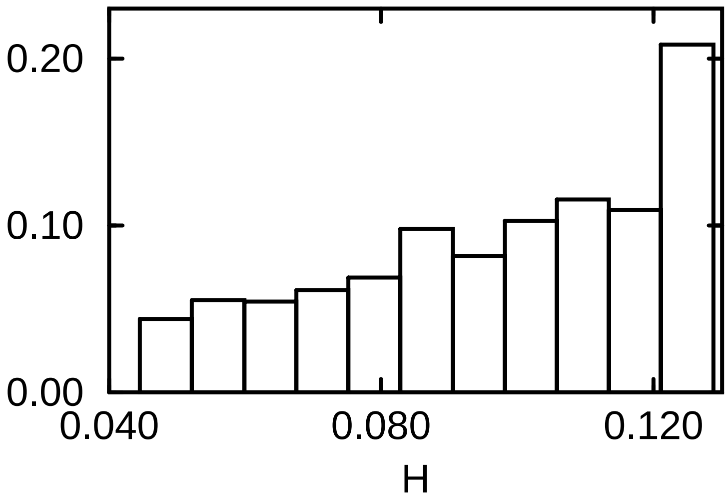

Figure 4.40: The histogram of the mean curvature for the surface DDU

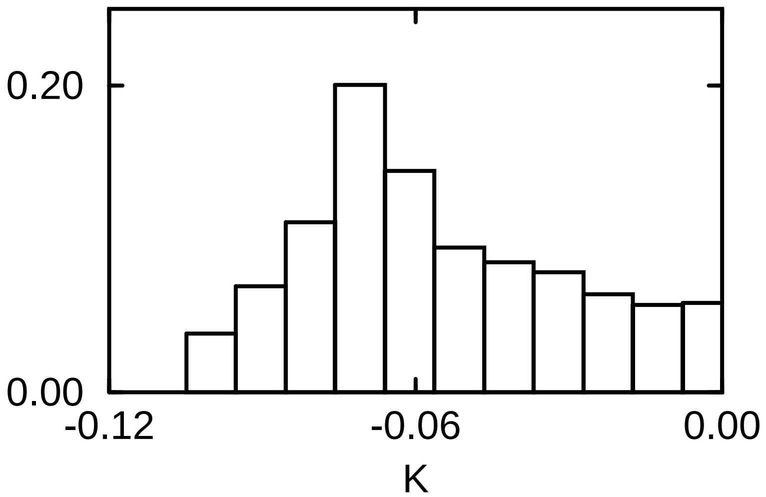

Figure 4.41: The histogram of the Gaussian curvature for the surface DDU

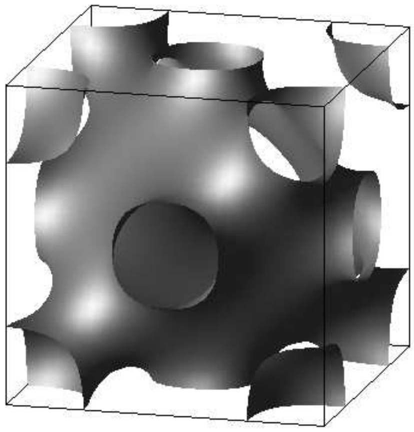

Figure 4.42: GM surface generated from the functional (2.9). The unit cell. Off front view.

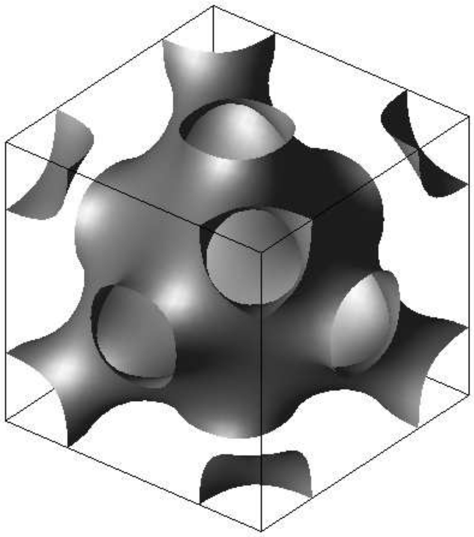

Figure 4.43: GM surface generated from the functional (2.9). The unit cell. Off diagonal view.

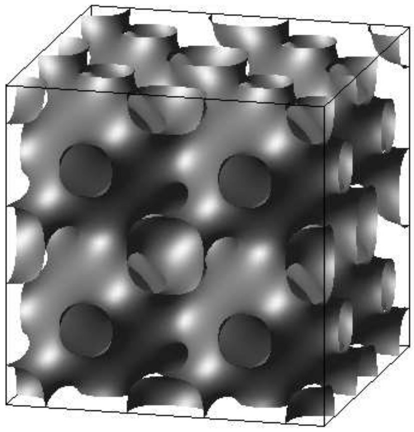

Figure 4.44: GM surface generated from the functional (2.9). 8 unit cells. Off front view.

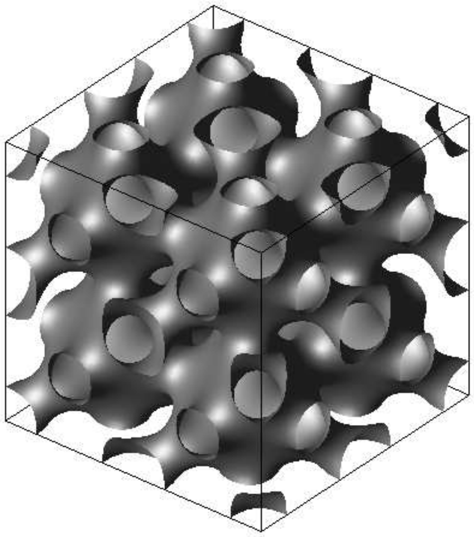

Figure 4.45: GM surface generated from the functional (2.9). 8 unit cells. Off diagonal view.

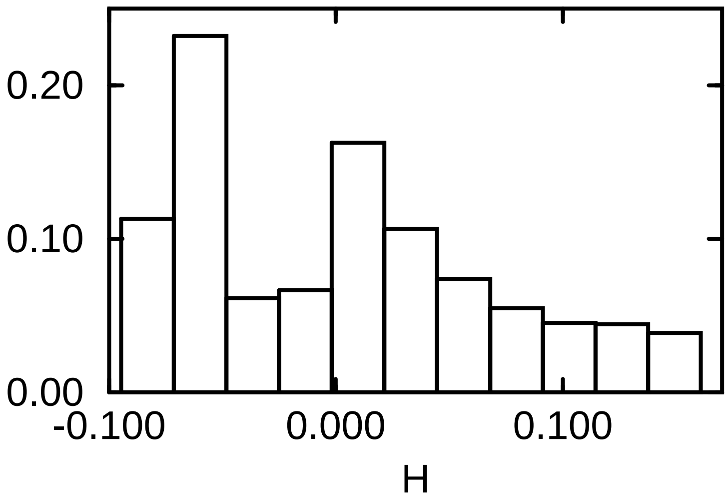

Figure 4.46: The histogram of the mean curvature for the surface GM

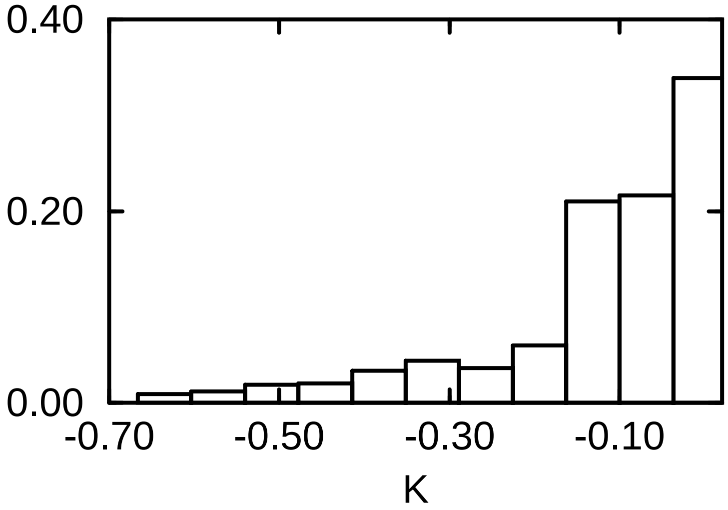

Figure 4.47: The histogram of the Gaussian curvature for the surface GM

Name	**Cell Length**	**Energy**	**Surface Area**	**Genus**	**Volume Fraction**
CPD	15.2	-0.174	4.3588	14	0.515
BFY	16.7	-0.180	4.9641	19	0.5
DDU	23.04	-0.181	3.2972	3	0.529
GM	24.00	-0.176	3.4925	6	0.520
GP	18.32	-0.183	5.4840	21	0.5

Table 4.2: The geometrical properties of low genus surfaces obtained from the functional (2.9), for the parameters $f_0 = 0.0$, $g_0 = -3.0$. Surface area $\tilde{S}$, in the table, is the normalized per face of the unit cube L^2 surface area S of the interface in the unit cell, $\tilde{S} = S/L^2$, $L = (N-1)h$. The energy is given per unit volume.

many of these surfaces are minimal. However, the definite answer is left to mathematicians, which have investigated the problem of minimal surfaces for over 100 years. We are more interested in the physics of the problem and in general in obtaining new undiscovered periodic surfaces (not necessarily minimal). Certainly not all of the generated surfaces are minimal e.g. GP, DDU, GM (see the mean curvature histograms).