4.3 High genus embedded periodic surfaces

The high genus surfaces are most easily generated for the values of parameters f_0 and g_0 in (2.9) taken near the boundary of lamellar and microemulsion phase and for the bigger unit cell length, $L = (N-1)h$, than the length of structures of low genus. This suggest the the microemulsion can be considered as a bicontinuous structure of high genus surface. These are only speculations based on theoretical studies. New experiments are needed to check these speculations. We hope that our results will help experimentalists to design such experiments.

We present in this chapter several examples of high genus surfaces of different symmetry. The structure CD has the symmetry of double diamond phase, the structure SCN1 has the symmetry of the simple cubic phase, and the structures GX1,GX2, GX3, GX5, GX6, GX7 have the gyroid phase symmetry.

The CD structure is especially interesting, because it has the same symmetry as the Schwartz diamond D phase. It has been generated in the same way as the diamond D phase, that is we have reduced the unit cell by a factor 1/192. Having done such a reduction we hardly expected the possibility of generation of a new surface. In fact, enlarging the length of the cubic cell we usually have got the multiple replicas of the diamond D structure, except the case when CD structure was generated.

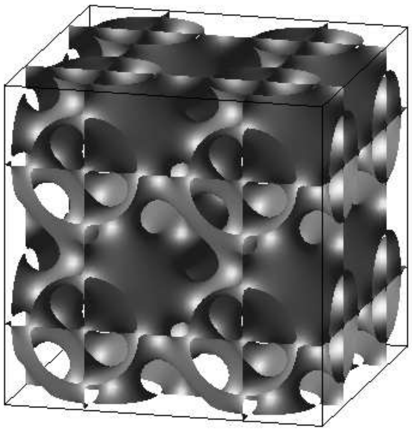

Figure 4.48: CD surface generated from the functional (2.9). The unit cell. Off front view.

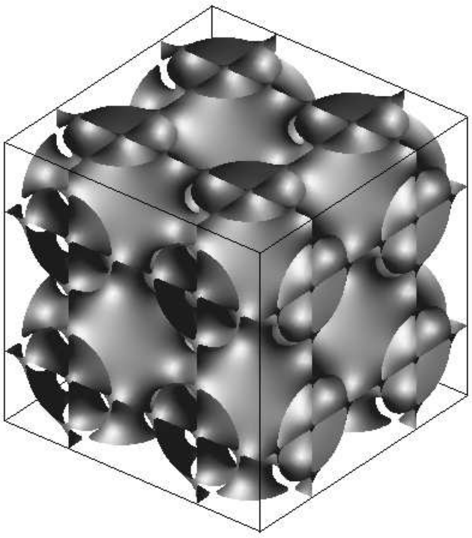

Figure 4.49: CD surface generated from the functional (2.9). The unit cell. Off diagonal view.

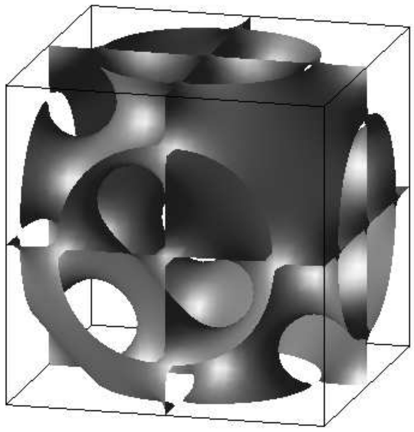

Figure 4.50: CD surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

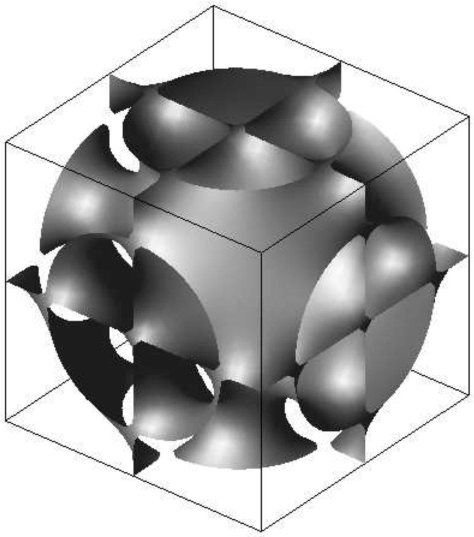

Figure 4.51: CD surface generated from the functional (2.9). 1/8 of the unit cell. Off diagonal view.

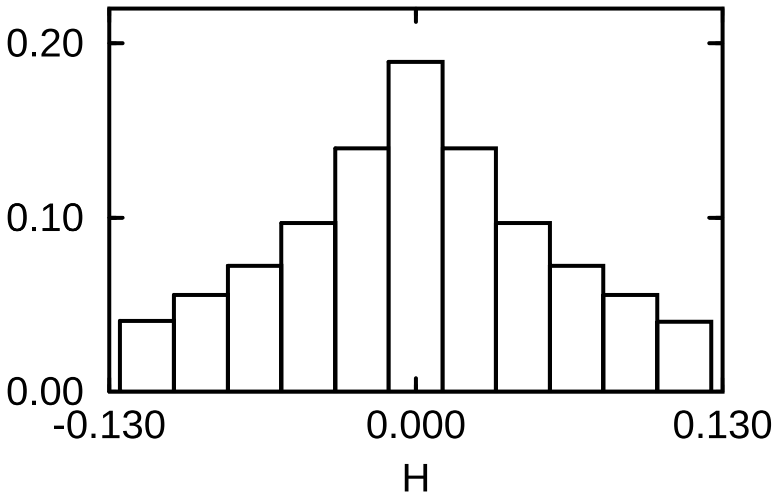

Figure 4.52: The histogram of the mean curvature for the surface CD

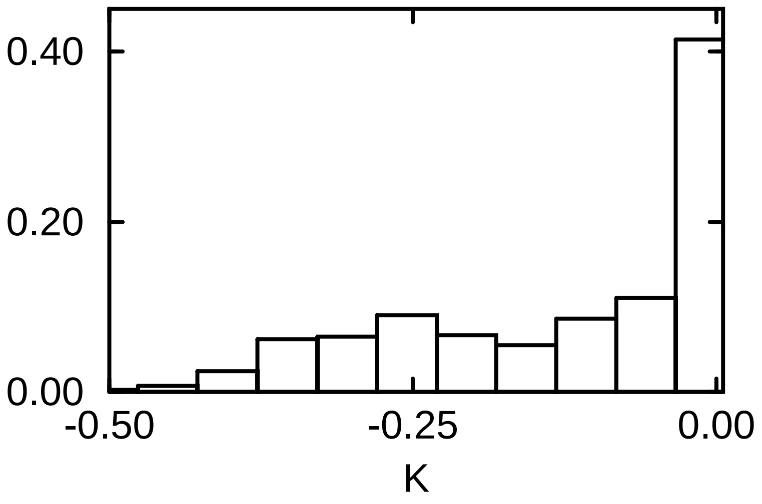

Figure 4.53: The histogram of the Gaussian curvature for the surface CD

The SCN1 surface is similar to BFY surface, but because its unit cell length is larger the surface is more complex. Comparison of these surfaces can be used as an example how the functional (2.9) keeps the size of water and oil region the same for different structures. For the bigger cell length the surface dividing the oil and water region is folded few times in order to keep the sizes of the oil and water region resulted from the functional (2.9) for a given values of the parameters f_0, g_0. It is amazing that the surface, representing the layer of surfactant in the model (2.9), is folded in such a way that its mean curvature tends to be zero.

The wealth of the high genus structures, even for the same symmetry, suggest the possibility of coexistence of these structures. We can speculate on the structure of microemulsion on the basis of these results. One can imagine that microemulsion is locally ordered like the phases we describe. Thus it is structured but globally disordered.

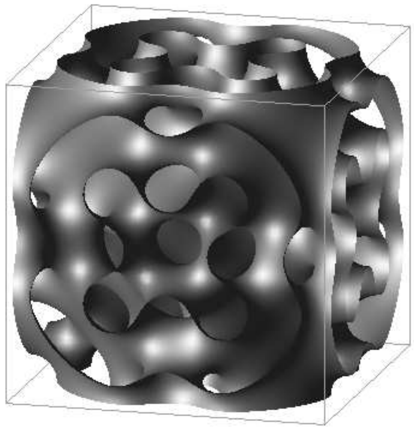

Figure 4.54: SCN1 surface generated from the functional (2.9). The unit cell. Off front view.

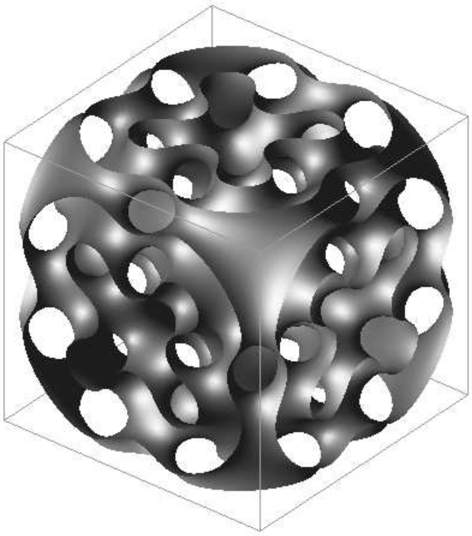

Figure 4.55: SCN1 surface generated from the functional (2.9). The unit cell. Off diagonal view.

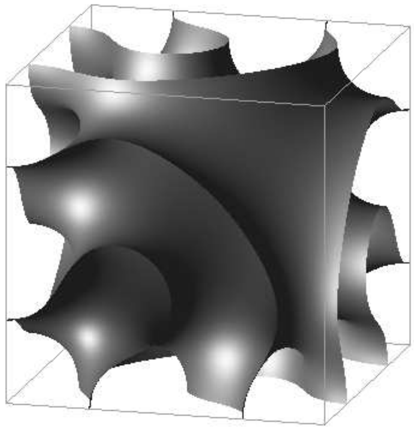

Figure 4.56: SCN1 surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

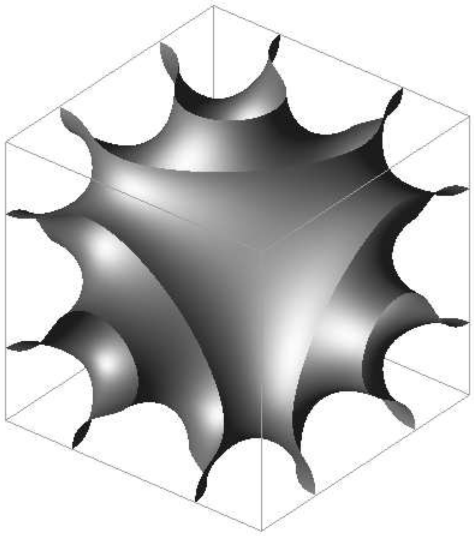

Figure 4.57: SCN1 surface generated from the functional (2.9). 1/8 of the unit cell. Off diagonal view.

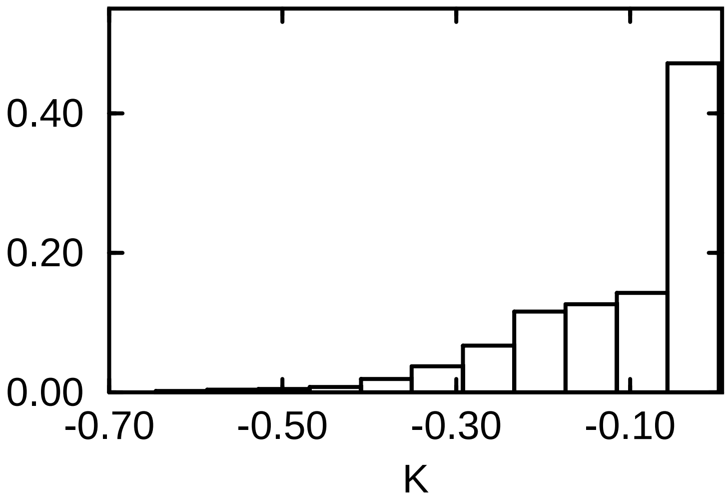

Figure 4.58: The histogram of the Gaussian curvature for the surface SCN1

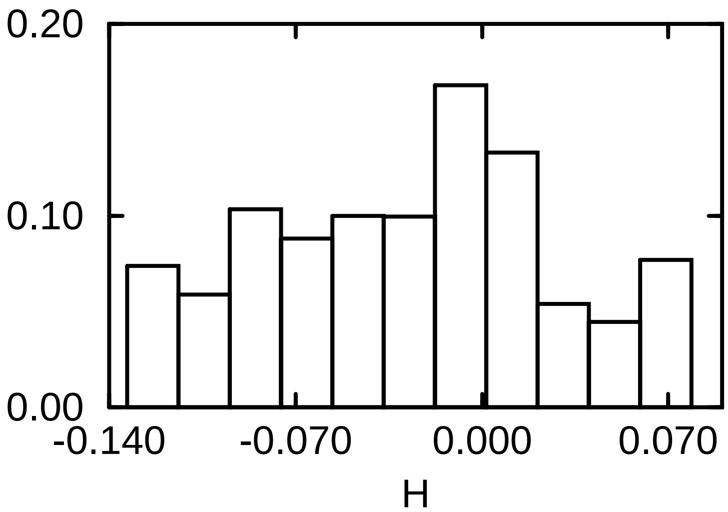

Figure 4.59: The histogram of the mean curvature for the surface SCN1