We have generated high genus surfaces of vanishing curvatures. The existence of such surfaces in real systems was questioned. The researches argued that it is impossible to build high genus periodic surface because of large curvatures, which could not be accommodated by the displacement of surfactant molecules at the surface [59]. As we can see from Table 4.3 high genus surfaces have large sizes of the unit cell and therefore their curvatures (see Gaussian curvatures) are similar in magnitude to those of surfaces of low genus. Therefore the argument against the existence of these surfaces does not hold.

The pictures of the high genus structures, especially the gyroid ones, strongly resemble the pictures of microemulsion taken during freeze fracture microscopy studies [5, 8]. The gyroid high genus surfaces have in general lower free energy that the surfaces of other symmetries. We can speculate that this symmetry would be preferred in real systems. In fact the Schoen gyroid G minimal surface is the most common in nature among the known minimal surfaces. The gyroid surfaces do not have planes of symmetry. This may cause easier adaptation of their shape to the structures encountered in nature, like diblock copolymers, lipid-water solution or surfactant mixtures.

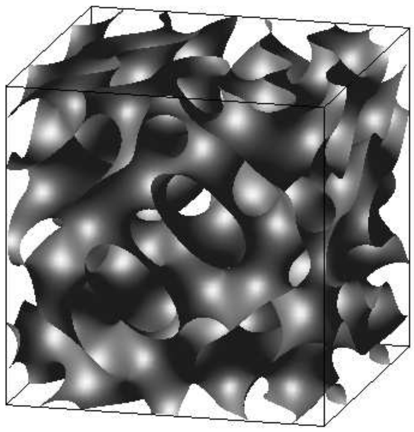

Figure 4.60: GX1 surface. The unit cell. Off front view.

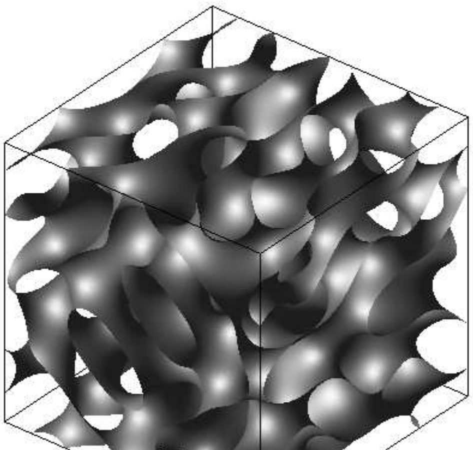

Figure 4.61: GX1 surface. The unit cell. Off diagonal view.

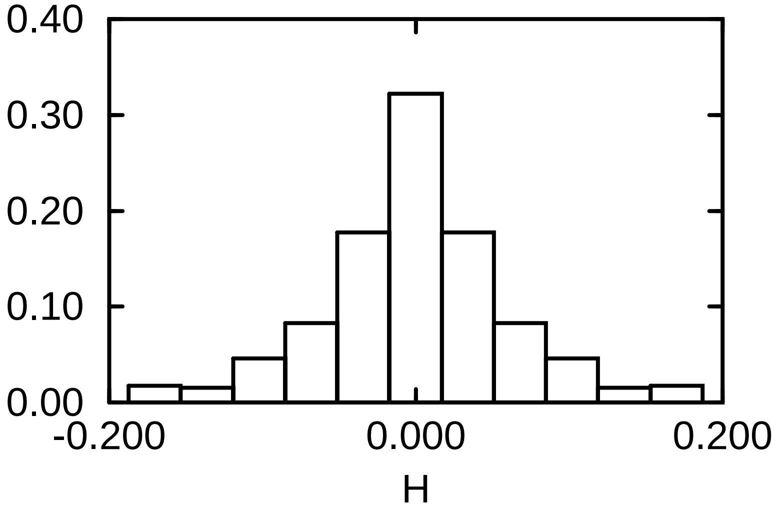

Figure 4.62: The histogram of the mean curvature for the surface GX1

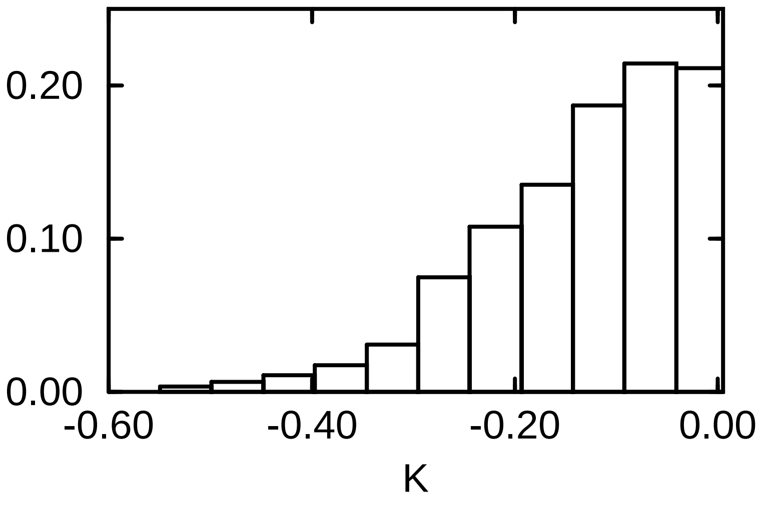

Figure 4.63: The histogram of the Gaussian curvature for the surface GX1

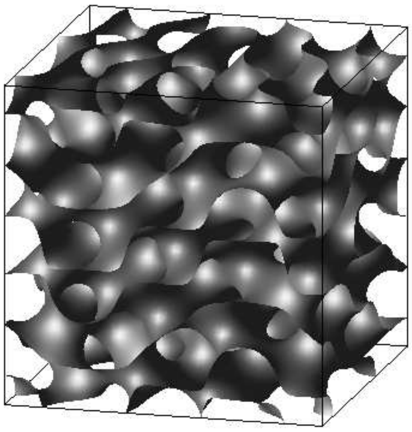

Figure 4.64: GX2 surface. The unit cell. Off front view.

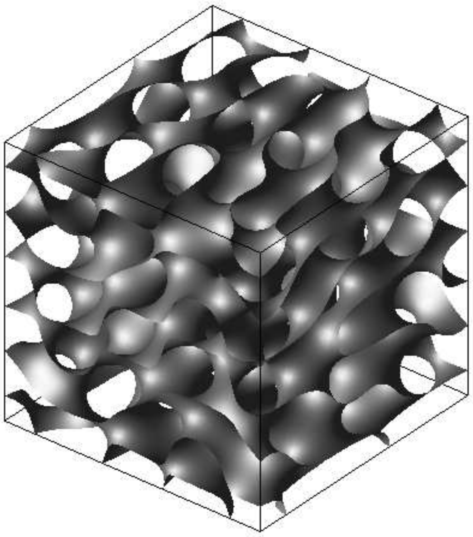

Figure 4.65: GX2 surface. The unit cell. Off diagonal view.

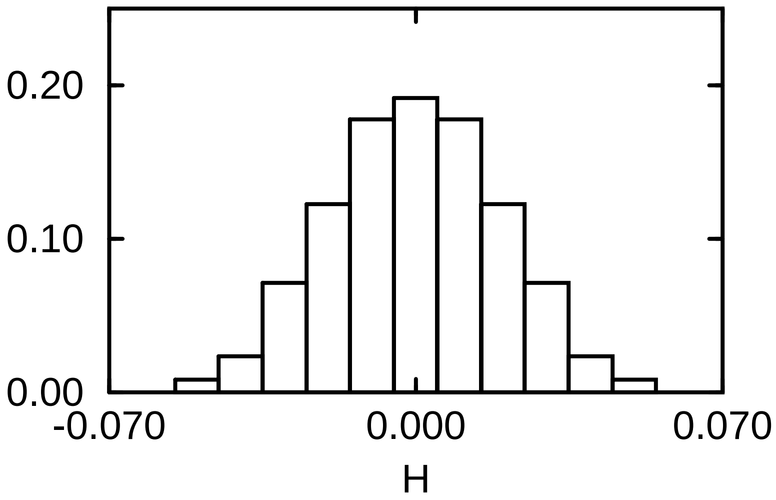

Figure 4.66: The histogram of the mean curvature for the surface GX2

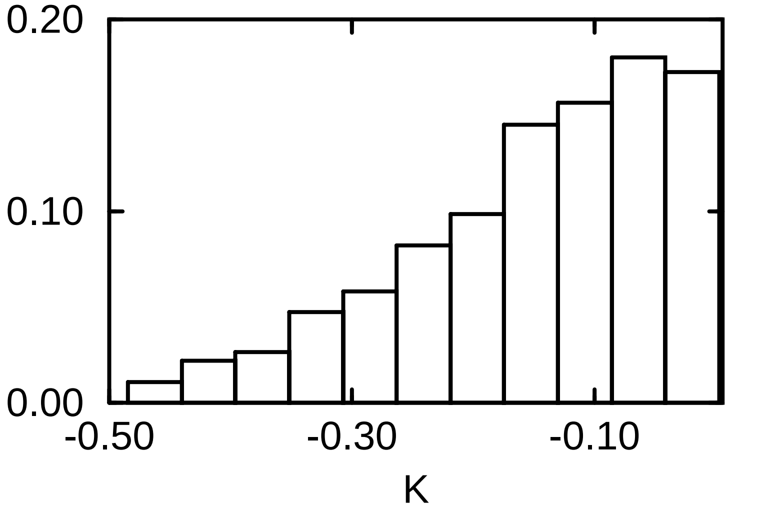

Figure 4.67: The histogram of the Gaussian curvature for the surface GX2

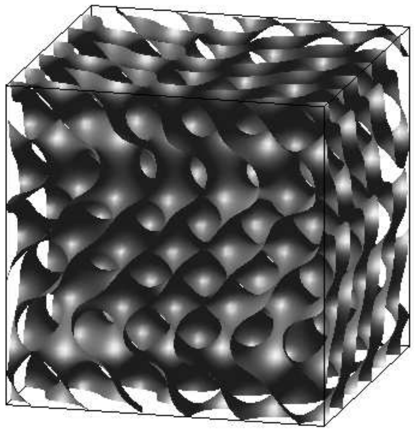

Figure 4.68: GX3 surface. The unit cell. Off front view.

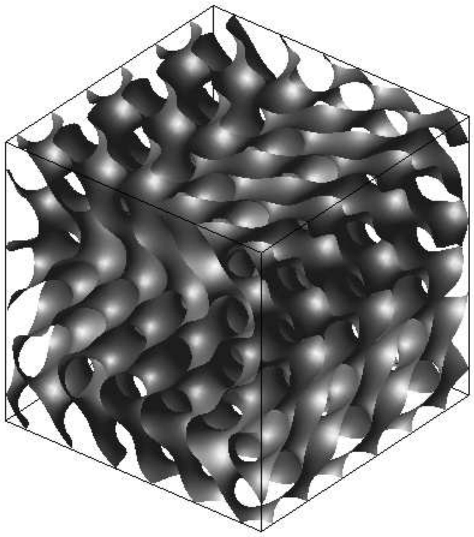

Figure 4.69: GX3 surface. The unit cell. Off diagonal view.

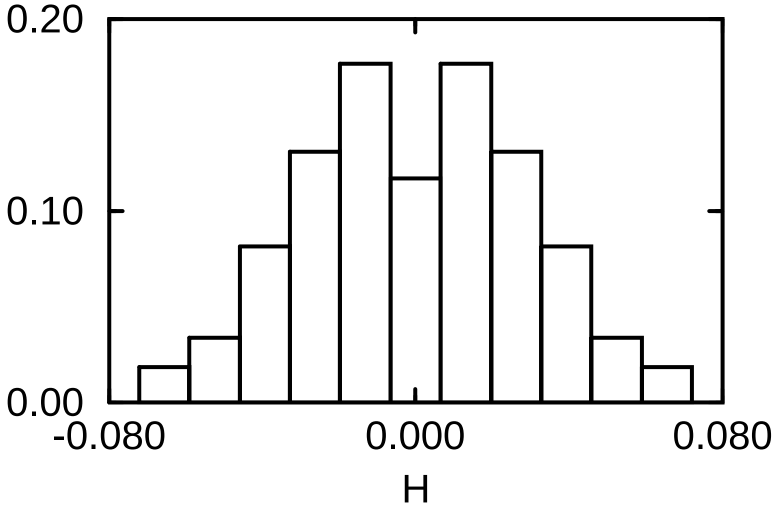

Figure 4.70: The histogram of the mean curvature for the surface GX3

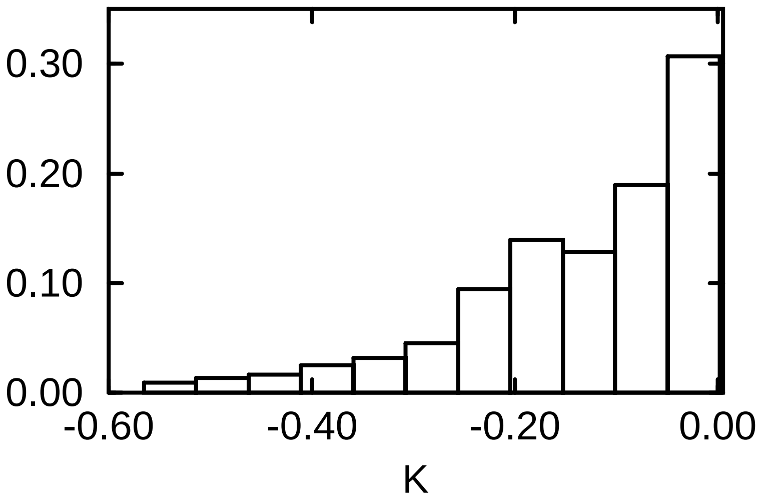

Figure 4.71: The histogram of the Gaussian curvature for the surface GX3

The properties of high genus surfaces are calculated with lower accuracy than the properties of those of low genus ones. It is so because we have used the same size of the lattice, N, for the unit cell. This results in bigger lattice spacing, h. For high genus structures more surface has to be accommodated in the unit cell, therefore the calculation of the derivatives is less accurate.

The biggest errors are in calculations of the curvatures, because all possible approximations are accommodated in these calculations. First is the approximation used to find the surface $\phi(\mathbf{r}) = 0$. The location of this surface has to be linearly interpolated between the lattice sites. The folmulas for the mean and Gaussian curvature are also numerical approximations of the analytic expressions. The derivatives used in calculations are also numerical approximations. The points used to calculate the derivatives have to be interpolated between the lattice sites. Finally the discretization and minimization of the functional also introduces some errors. It is amazing that, in spite of all those sources of errors, the curvatures are calculated with such a high accuracy, which can be seen looking at the histograms of the mean curvature for P,D,G minimal surfaces.

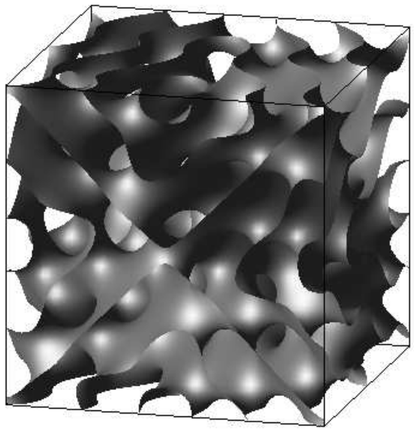

Figure 4.72: GX4 surface. The unit cell. Off front view.

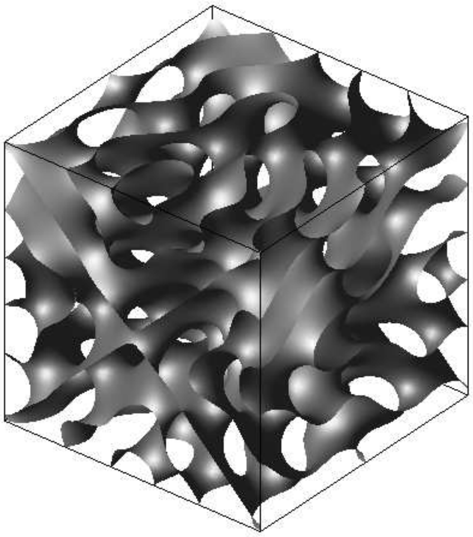

Figure 4.73: GX4 surface. The unit cell. Off diagonal view.

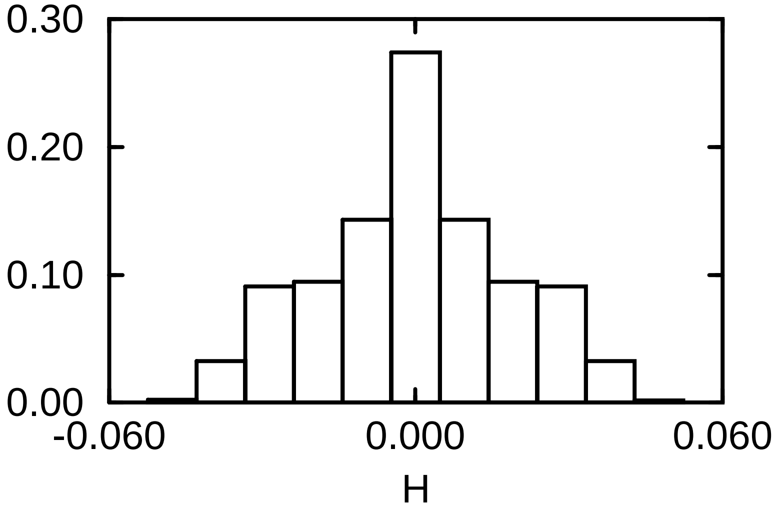

Figure 4.74: The histogram of the mean curvature for the surface GX4

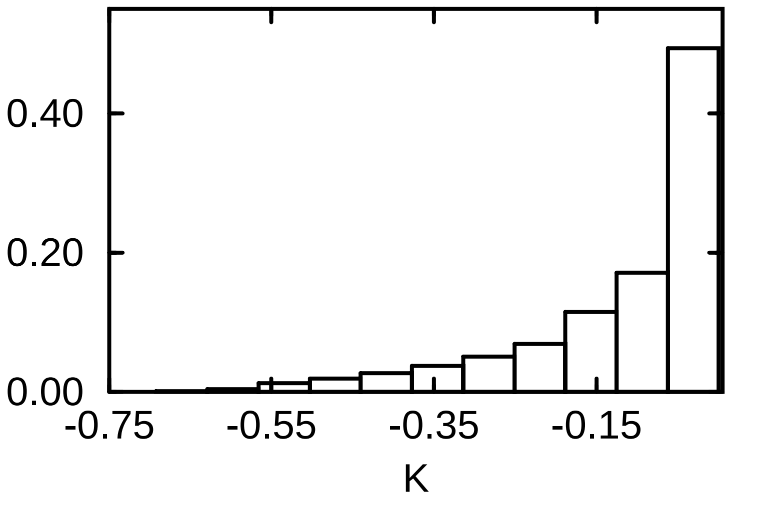

Figure 4.75: The histogram of the Gaussian curvature for the surface GX4

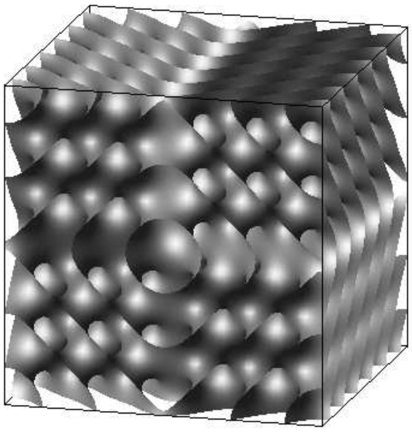

Figure 4.76: GX5 surface. The unit cell. Off front view.

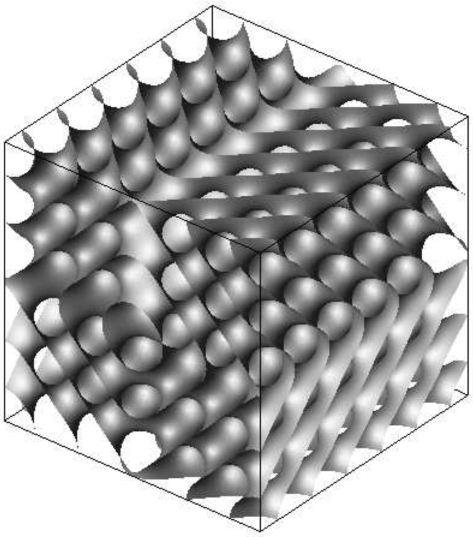

Figure 4.77: GX5 surface. The unit cell. Off diagonal view.

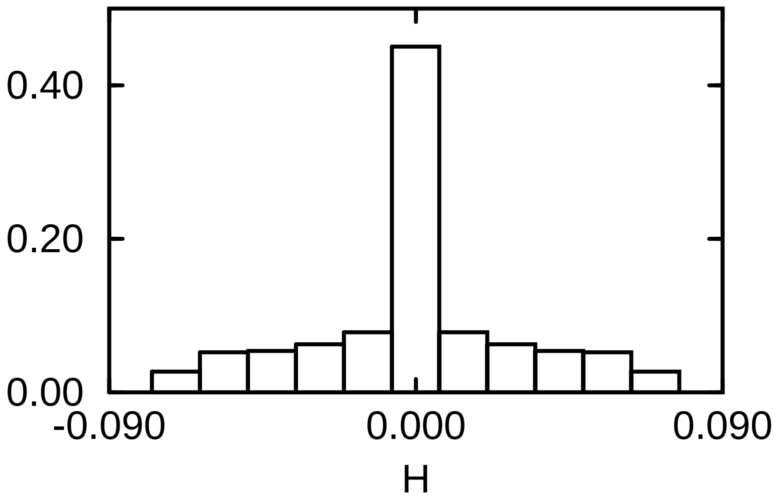

Figure 4.78: The histogram of the mean curvature for the surface GX5

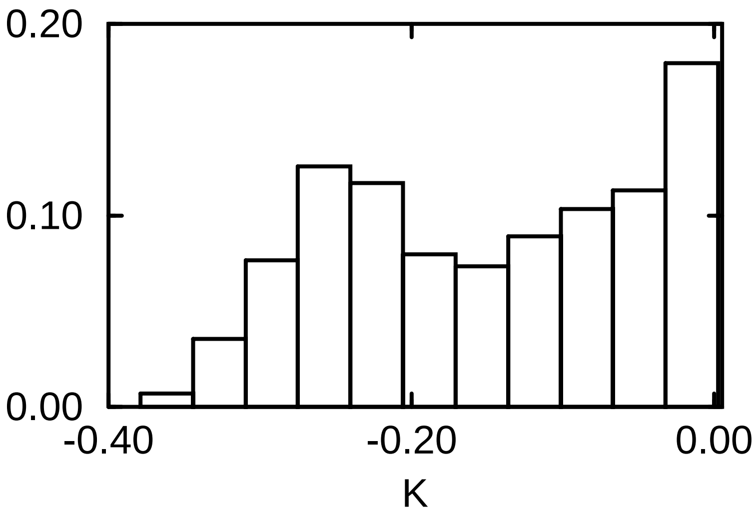

Figure 4.79: The histogram of the Gaussian curvature for the surface GX5

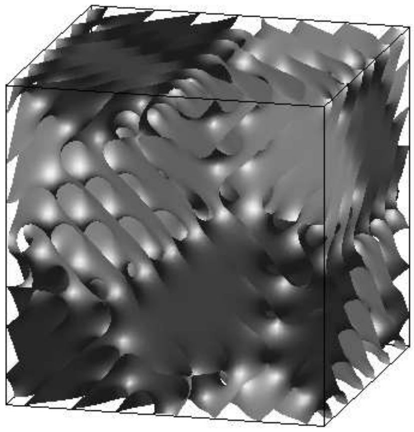

Figure 4.80: GX6 surface. The unit cell. Off front view.

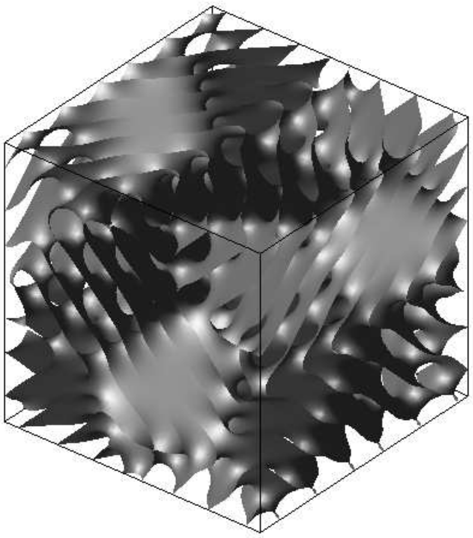

Figure 4.81: GX6 surface. The unit cell. Off diagonal view.

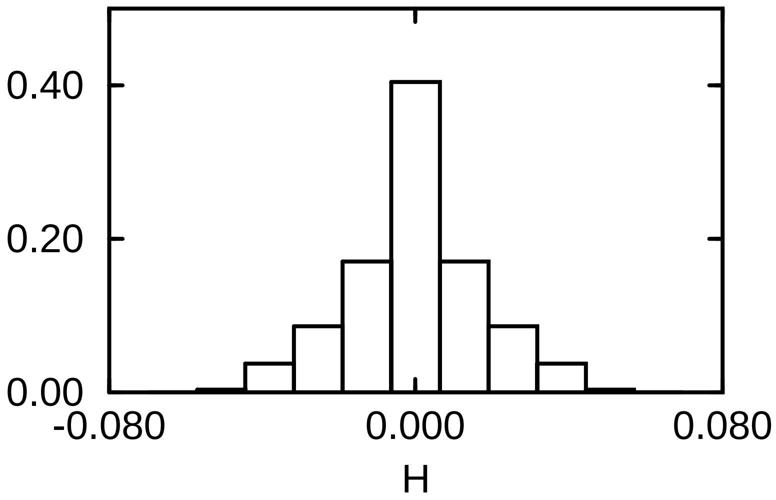

Figure 4.82: The histogram of the mean curvature for the surface GX6

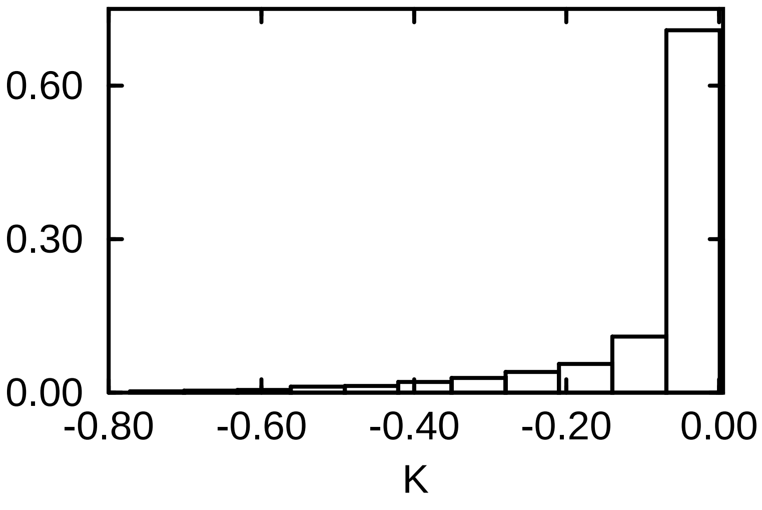

Figure 4.83: The histogram of the Gaussian curvature for the surface GX6

Name	Cell Length	Energy	Surface Area	Genus
SCN1	25.6	-0.178	7.4288	45
CD	28.88	-0.167	8.2257	73
GX1	25.36	-0.182	7.622	53
GX2	26.48	-0.183	8.081	69
GX3	31.72	-0.181	9.657	109
GX4	26.16	-0.186	7.907	53
GX5	34.40	-0.178	10.519	157
GX6	41.32	-0.186	12.459	141

Table 4.3: The geometrical properties of high genus surfaces obtained from the functional (2.9), for the parameters $f_0 = 0.0$, $g_0 = -3.0$. Surface area $\tilde{S}$, in the table, is the normalized per face of the unit cube L^2 surface area S of the interface in the unit cell, $\tilde{S} = S/L^2$, $L = (N-1)h$. The energy is given per unit volume. The volume fraction for all structures is 0.5.