4.4 Multiply continuous embedded periodic surfaces

The possibility of the existence of multiply continuous structures was never discussed so far to the best of our knowledge. Although the idea seems to be trivial nobody has considered it. So far only bicontinuous phases have been considered both in the theoretical and experimental studies. In the bicontinuous structure the single surface separates the volume into two disjoint subvolumes. In our phases there is more than one periodic surface which disconnects the volume into three or more disjoint subvolumes.The experimental results do not rule out the existence of such structures, contrary the experiment on the electrical conductivity support the idea [5, 6]. It would be very interesting to design an experiment answering the question whether the structures considered so far as bicontinuous are only bicontinuous or maybe they are multiply continuous.

The multiply continuous structures are most easily generated for the values of parameters f_0 and g_0 in (2.9) taken near the boundary of lamellar and water (oil) phase and for the bigger unit cell length, $L = (N-1)h$, than the length of structures of low genus.

We present the multiply continuous structures of the simple cubic phase symmetry (SCL1,SCL2, Figs. 4.84,4.86) and of the gyroid phase symmetry (GL1,GL2 Figs. 4.88,4.90). The SCL1 structure is triply continuous, the GL1,SCL2 are four-tuply continuous, and GL2 is six-tuply continuous. For bigger size of the unit cell one is able to generate the structures n-tuply continuous. It is remarkable that the volume fraction of oil and water is 0.5 for all these structures. The genera for all surfaces in a given structure are the same. Other properties of the multiply continuous structures are described in Table 4.4.

The SCL1 surface is especially interesting. Although the outer and inner surface looks different, see Fig. 4.84, they have the same surface area, see Table 4.4. In fact they are built of the same piece of the surface. The picture of 1/8 of the unit cell, see Fig. 4.85, explains how two different periodic surfaces can be built of the same surface patch.

The SCL2 structure is composed of three different embedded periodic surfaces, Figs. 4.86,4.87. The middle surface is the Schwarz minimal surface P, its normalized surface area of these surfaces are the same within the numerical errors, see Tables 4.4, 4.1. Similar, the middle phase surface in GL1 structure is the Schoen minimal surface G.

The GL2 structure suggest that one can generate arbitrary n-tuply continuous structures. It is only necessary to set the cell length sufficiently large. We have not attempted generation of such structures because, due to the limits imposed by computer memory and speed of processor, the lattice spacing would be too big for a given size of the lattice to obtain reasonable accuracy.

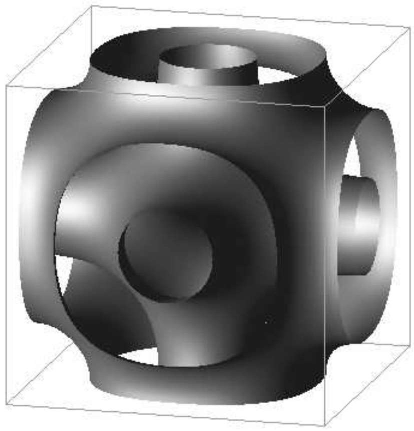

Figure 4.84: SCL1 surface generated from the functional (2.9). The unit cell. Off front view.

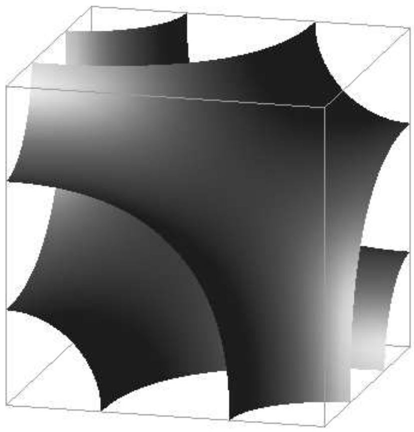

Figure 4.85: SCL1 surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

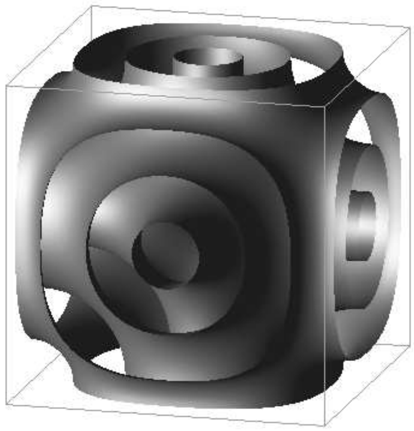

Figure 4.86: SCL2 surface generated from the functional (2.9). The unit cell. Off front view.

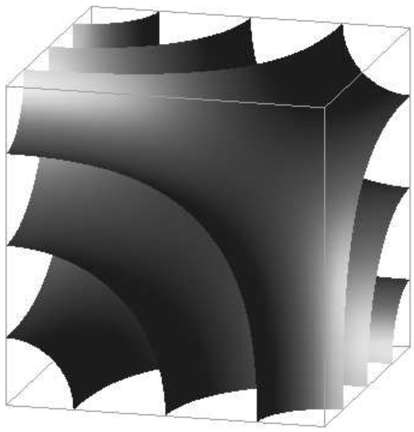

Figure 4.87: SCL2 surface generated from the functional (2.9). 1/8 of the unit cell. Off front view.

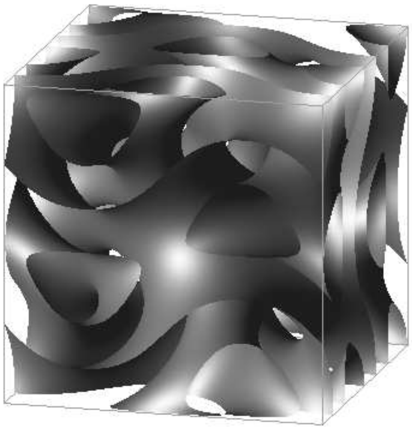

Figure 4.88: GL1 surface generated from the functional (2.9). The unit cell. Off front view.

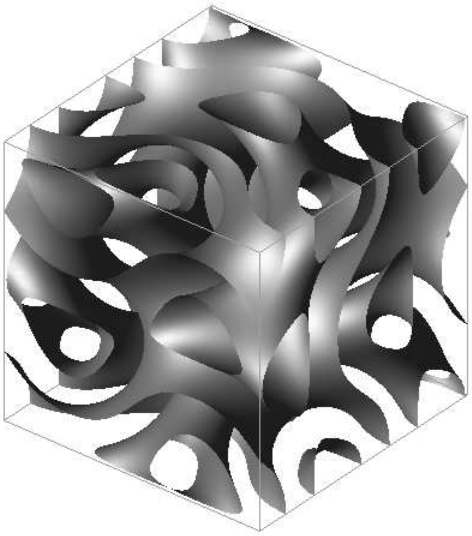

Figure 4.89: GL1 surface generated from the functional (2.9). The unit cell. Off diagonal view.

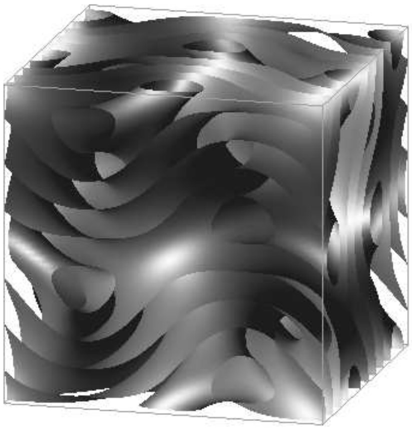

Figure 4.90: GL2 surface generated from the functional (2.9). The unit cell. Off front view.

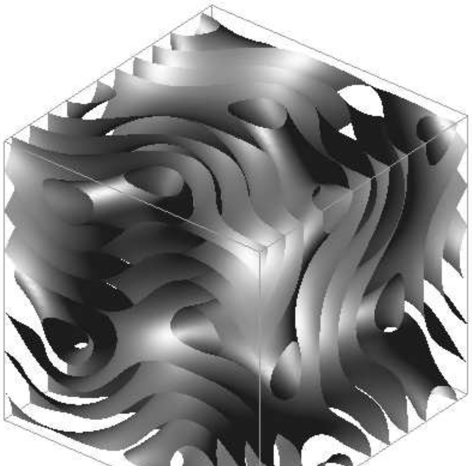

Figure 4.91: GL2 surface generated from the functional (2.9). The diagonal cell. Off front view.

Name	Cell Length	Energy	Surface Area	Genus
SCL1	14.96	-0.175	4.090	3
1			2.045	
2			2.045	
SCL2	21.14	-0.178	5.780	3
1			1.716	
2			2.348	
3			1.716	
GL1	26.32	-0.187	7.546	5
1			2.226	
2			3.096	
3			2.226	
GL2	41.16	-0.185	11.887	5
1			1.659	
2			2.736	
3			3.097	
4			2.736	
5			1.659	

Table 4.4: The geometrical properties of multiply continuous structures obtained from the functional (2.9), for the parameters $f_0 = 0.0$, $g_0 = -3.0$. Surface area $\tilde{S}$, in the table, is the normalized per face of the unit cube L^2 surface area S of the interface in the unit cell, $\tilde{S} = S/L^2$, $L = (N-1)h$. The energy is given per unit volume. The volume fraction for all surfaces is 0.5. The genus is given for any single surface in the structure.

4.5 Stability of different phases in the model

We have investigated many bicontinuous phases of different symmetries, genera and dimensions of the unit cell. The bicontinuous phase with the lowest value of the free energy turned out to be the gyroid, but the only stable liquid crystalline phase in this model is the lamellar phase. Fig. 4.92 shows the plot of the free energy as the function of the parameter f_0 for the lamellar and gyroid phase. The phase diagram for the model (2.9), in mean-field

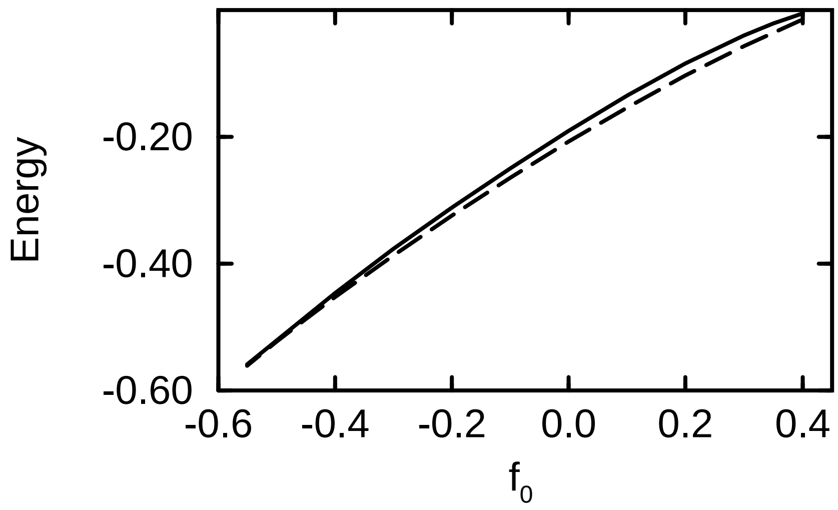

Figure 4.92: The free energy per unit volume for the lamellar phase (dashed line) and gyroid phase (solid line) for the model (2.9). The parameter g_0 was equal to -3.0.

approximation, calculated by Gompper and Zschocke [21] and checked here is shown in Fig. 4.93. Other bicontinuous phases behave in the same way

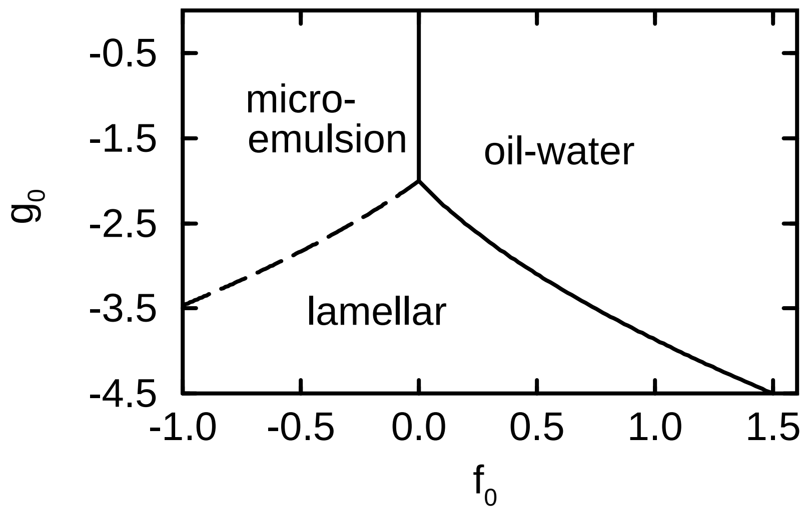

Figure 4.93: The phase diagram for the model (2.9)

as the gyroid and lamellar. The value of the free energy in the vicinity of microemulsion boundary converges to the value of the free energy for the microemulsion.

We have also studied the stability of bicontinuous phases for different function describing surfactant, $g[\phi(\mathbf{r})]$. We have used the following form of $g[\phi(\mathbf{r})]$:

$$g\big[\phi(\mathbf{r})\big] = g_2\phi(\mathbf{r})^4 + g_0. \tag{4.4}$$

We have expected that this form of $g[\phi(\mathbf{r})]$ would make the interface between oil and water sharper and therefore it would lower the free energy. The interface indeed was sharper as we expected, but the unit cell length decreased also. This resulted in higher values of the free energy than before and all bicontinuous phases were metastable as in the previous case. It might suggest that one order parameter functional is not sufficient to describe the behavior of ordered phases. Howver, it has been observed recently [60] that one may expect the stabilization of the gyroid phase in Monte Carlo simulation. Thus the fluctuations can play a significant role in the stabilization of bicontinuous phases. We note that in the case of multi-parameter Landau models introduced in recent years [5] we may expect the stabilization of the various phases which in the one order parameter Landau model are only metastable [25]. Our current results are a very good starting point for investigation of the stability of bicontinuous phases in multi-parameter Landau functionals.

Chapter 5

Summary and Conclusion

The method of solving the functional (2.9), we have applied, allowed us to make several discoveries. **We have discovered that the solutions of the physical model of microemulsion can be triply periodic minimal surfaces**. To the best of our knowledge this is the first such discovery. So far the triply periodic minimal surfaces were generated from the Weirstrass representation or from the definition of the mean curvature.

We have discovered many triply periodic surfaces of non-positive Gaussian curvature. Some of them may be new minimal surfaces. Especially interesting are the surfaces of high genus. The existence of such surfaces may suggest that microemulsion can be such a surface of very high genus. For over one hundred years mathematicians have discovered a few infinite embedded periodic minimal surfaces of cubic symmetry. **We have discovered**, by using our method, **several new infinite periodic surfaces of cubic symmetry**, which are very likly to be miminal. The problem of peridic surfaces is being rediscovered over recent years because of their importance in physich and biology. We are convinced that this work will help to make other workers signifacant progress in this area.

We have discovered, never considered yet, multiply continuous cubic structures formed in ternary mixtures of water, oil and surfactant. The idea of multiple continuity of cubic structures is the novel one. Our results call for new experimental techniques which could be used to discern between bicontinuous and multiply continuous structures. We hope that these new structures will soon be discovered in real systems.

Very often to see means to understand. We have not only calculated the properties of many cubic structures, investigated their stability, but also presented the pictures of new structures. We hope that this will help other workers better understand the phenomena in ternary liquid mixtures, diblock copolymers, and biological systems.

We have presented the general method for the generation of periodic surfaces of non-positive Gaussian curvature. This method can be well applied by physicists working in soft condensed matter, mathematicians working in topology, biologists and crystallographers. We are positive that the richness of the method is far from being explored by our work.

The answers we have given in this work do not close the problem we have investigated. Contrary, our results open new area to investigate by other workers, can be a pointer for experimentalists to design new experiments. Our results are also an ideal starting point, for theoreticians, to pursue the studies of Landau-Ginzburg models with more than one order parameter field [25], to investigate dynamical properties of complex systems [61, 62] or to study complex fluids in confined geometries [22]. These are only a few benefits of this work and we are convinced that this is not a complete list.

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